ESD-TR-90-326

MTR-10910

(2)

AD-A228 871

A Successive Partial-Relaxation Gaussian Algorithm for Area Navigation Operations with the Microwave Landing System (MLS)

By

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October 1990

Prepared for

Deputy Program Director
Communications and Airspace Management SPO
Electronic Systems Division
Air Force Systems Command
United States Air Force
Hanscom Air Force Base, Massachusetts

Project No. 5420

Prepared by

The MITRE Corporation
Bedford, Massachusetts

Contract No. F19628-89-C-0001

REVIEW AND APPROVAL

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UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION
Unclassified

1b. RESTRICTIVE MARKINGS

2a. SECURITY CLASSIFICATION AUTHORITY

2b. DECLASSIFICATION / DOWNGRADING SCHEDULE

3. DISTRIBUTION / AVAILABILITY OF REPORT
Approved for public release; distribution unlimited.

4. PERFORMING ORGANIZATION REPORT NUMBER(S)
MTR-10910
ESD-TR-90-326

5. MONITORING ORGANIZATION REPORT NUMBER(S)

6a. NAME OF PERFORMING ORGANIZATION
The MITRE Corporation

6b. OFFICE SYMBOL (If applicable)

7a. NAME OF MONITORING ORGANIZATION

6c. ADDRESS (City, State, and ZIP Code)
Burlington Road
Bedford, MA 01730

7b. ADDRESS (City, State, and ZIP Code)

8a. NAME OF FUNDING / SPONSORING ORGANIZATION
Deputy Program (continued)

8b. OFFICE SYMBOL (If applicable)
ESD/TG

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
F19628-89-C-0001

8c. ADDRESS (City, State, and ZIP Code)
Electronic Systems Division, AFSC
Hanscom AFB, MA 01731-5000

10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO	PROJECT NO.	TASK NO	WORK UNIT ACCESSION NO
	5420		

11. TITLE (Include Security Classification)
A Successive Partial-Relaxation Gaussian Algorithm for Area Navigation Operations with the Microwave Landing System (MLS)

12. PERSONAL AUTHOR(S)
Hatzis, Patricia M., Powell, Frederic D.

13a. TYPE OF REPORT
Final

13b. TIME COVERED
FROM ________ TO ______

14. DATE OF REPORT (Year, Month, Day)
1990 October

15. PAGE COUNT
45

16. SUPPLEMENTARY NOTATION

17. COSATI CODES

FIELD	GROUP	SUB-GROUP

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
Calculated Approach
Gauss
Gauss-Seidel
Microwave Landing System
Partial Relaxation
Relaxation

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Position reconstruction algorithms (PRAs) based on Gaussian techniques converge very slowly, or diverge, for some geometries of ground unit and aircraft location which are within MLS system coverage. On the other hand, algorithms based on Newton-Raphson techniques usually converge very rapidly but impose a significantly greater storage requirement and computational burden on the avionics. This report presents a modified Gaussian algorithm which uses a relaxation factor to achieve rapid convergence for all geometries, and with a computational burden much less than the equivalent Newton-Raphson algorithm. It presents the theoretical foundations of this algorithm and various results showing its effects. It also compares the algorithm's storage and computational burdens against Gaussian, rotated-coordinate Gaussian, and Newton-Raphson equivalent PRAs in the MLS context.

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT
☐ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT ☐ DTIC USERS

21. ABSTRACT SECURITY CLASSIFICATION
Unclassified

22a. NAME OF RESPONSIBLE INDIVIDUAL
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22c. OFFICE SYMBOL
Mail Stop D135

DD Form 1473, JUN 86 Previous editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE
UNCLASSIFIED

a. Director, Communications and Airspace Management SPO.

EXECUTIVE SUMMARY

This report, which addresses area navigation with the Microwave Landing System (MLS), presents a Cartesian coordinate position reconstruction algorithm (PRA) which uses the principle of Gauss-Seidel (GS) successive partial relaxation (SPR). This SPR PRA has an even lower storage requirement and computational burden than its predecessor, which used the principle of GS coordinate rotation [1]. Like its predecessor, this SPR PRA is stable and converges rapidly throughout the MLS maximum lateral coverage volume (±60°), whereas the PRAs published in reference 2 were shown in reference 4 to diverge or exhibit false solutions within the MLS minimum lateral coverage of ±40°. This report presents this new SPR PRA, develops its theoretical foundations, and compares its various characteristics with other PRAs in the MLS context.

The GS procedure, originally intended for solving sets of linear equations, is highly appropriate for solving nonlinear sets in which each equation can be solved explicitly for a different variable, as in the MLS situation. In conventional GS iteration, equation "1" is solved for variable "1" using initial values for the other variables, then equation "2" is solved for variable "2" using the new value for variable "1", and so on. The method of partial relaxation first determines a "new" value for variable "2", by the procedure just described, then determines the increment which that new value represents, multiplies that increment by a relaxation-factor $0 < \omega < 2$, and finally replaces the "new" value by the sum of the increment and the value of variable "2" from the prior iteration. If $0 < \omega < 1$, the practice is called "under-relaxation,", if $1 < \omega < 2$, it is called "over-relaxation", and if $\omega = 1$ the conventional GS result is recovered. "Under-relaxation" is appropriate to the MLS problem, and adjusting the value of ω during iteration is the key element in the success of the SPR PRA in this report.

ACKNOWLEDGEMENT

This document has been prepared by The MITRE Corporation under Project No. 5420, Contract No. F19628-89-C-0001. The contract is sponsored by the Electronic Systems Division, Air Force Systems Command, United States Air Force, Hanscom Air Force Base, Massachusetts 01731-5000.

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SECTION 1

INTRODUCTION

The international civil aviation community will replace the Instrument Landing System with the Microwave Landing System (MLS) during the next decade; see reference 1. The MLS enables much more precise determination of the aircraft's location, and has much wider coverage. These added capabilities will enable conduct of area navigation (RNAV), including curved approaches, in the vicinity of the airport. In addition, it will not be necessary to locate the MLS azimuth antenna on the runway centerline; offset sites and therefore offset runways can be used, or the same set of ground units can service several runways. These new capabilities are accompanied by new complexities.

In RNAV or computed centerline operation with the MLS, it is necessary, given the data which define the sites of the three ground units, and the observations of azimuth angle, elevation angle and distance measuring equipment (DME), to determine in the avionics the aircraft position in Cartesian coordinates relative to the centerline of the desired runway. This process is not simple when the ground units are not collocated, and iteration is required to reconstruct the aircraft position from the data and observations. Two general types, or classes, of algorithms have been suggested for this purpose, Gaussian and Newton-Raphson; see reference 2. Gaussian position reconstruction algorithms (PRAs) are attractive as they tend to have small storage and computational burdens, in comparison to Newton-Raphson PRAs. However, they also may have divergence or slow convergence problems, depending on the particular algorithm and the geometry of the situation. Divergence has been observed within the system's minimum required lateral coverage of $\pm 40°$. On the other hand, Newton-Raphson PRAs may have singularities that preclude use in various regions of the MLS coverage.

This paper presents a Gaussian PRA which uses a partial relaxation to speed convergence. The PRA conditions the problem so that iteration is stable and rapid everywhere within the MLS coverage. It is less compact than the equivalent Gaussian algorithm and has a greater computational burden; however, it converges faster and converges within the maximum MLS lateral coverage volume of $\pm$ 60°. The algorithm converges more slowly than the equivalent Newton-Raphson PRA; however, it is more compact, has no singularities, and has a significantly-reduced computational burden.

Section 2 presents a heuristic discussion of the principles of the fastest member of the class of Gaussian algorithms, Gauss-Seidel iteration (GSI). In a simple and general way the section shows the operating principles of the GSI process, and motivates the concept of partial relaxation in this application. Section 3 provides the notation and mathematical foundation of GSI algorithms, section 4 develops and shows the behavior of a conventional GSI MLS PRA, and section 5 extends section 4 to include the partial relaxation principle and thus forms a successive partial-relaxation

(SPR) Gauss-Seidel iterative PRA. Comparisons of speed, computational burden, and storage requirements against a GSI, a rotated GSI and a Newton-Raphson PRA are presented in section 6, followed by conclusions. Appendices give the computer code for the proposed algorithm and a small but representative sample of the behavior of this algorithm for a variety of ground unit geometries and aircraft locations.

SECTION 2

PRINCIPLES OF GAUSS-SEIDEL ITERATION

This section presents a heuristic discussion of the principles of GSI. Its purposes are to provide an intuitive, although rudimentary, understanding of GSI, and, especially, to encourage the application of the relaxation factor principle used in this study.

Consider a pair of nonlinear functions, f and g, of x and y. The functions are continuous in the vicinity of their unique solution, and it is assumed that the magnitude of the slope of f with respect to x is small compared to that of g. This situation is shown in figure 2-1. An initial condition, x_0, is assumed for x, and f is evaluated with this value, yielding y_1. The other equation, g, is then evaluated with y_1, yielding x_1 which is in turn used in f to find y_2 etc. As iteration continues, the successive estimates of x and y converge to the solution. The process is shown by arrows in the figure.

Now consider the effect if the slope of f is appreciably greater, as in figure 2-2. It is evident that the process converges much more slowly than in figure 2-1.

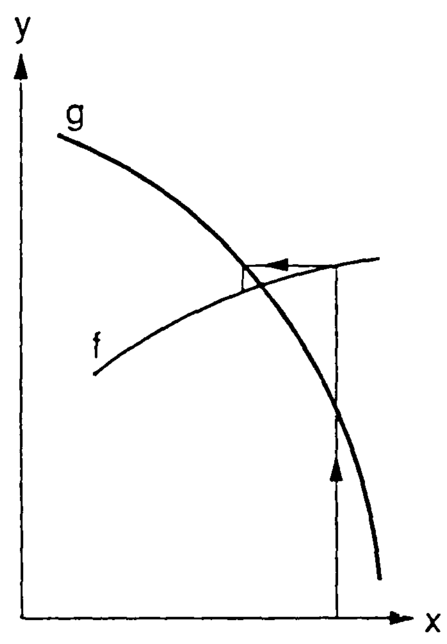

Figure 2-1. Fast Convergence

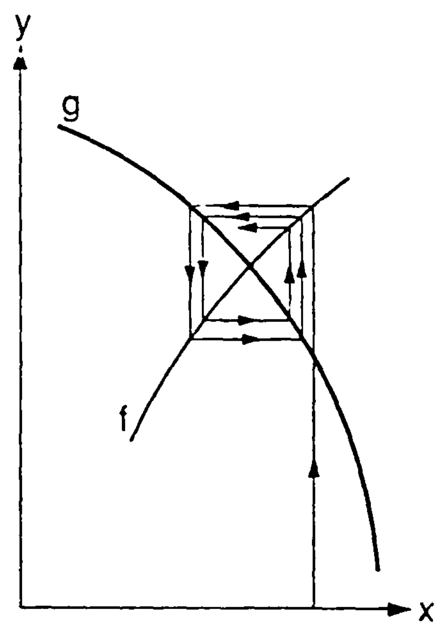

Figure 2-2. Slow Convergence

Some important principles of GSI may be deduced from these figures:

a. The increments to x and y are parallel to the x and y axes, and extend the complete distance from f to g, or from g to f;

b. Convergence slows as the magnitude of the slope of f approaches the magnitude of the slope of g;

c. New values of x and y are used as soon as they are available.

Regarding the GSI method described above, the partial relaxation principle is now considered. A moderated increment, or partial relaxation, can speed the slow convergence shown in figure 2-2. Standard GSI produces a new value $x_{k+1} = g^{-1}[f(x_k)]$. The relaxation factor is called ω and the new value produced is $x_{k+1} = x_k + \omega\{ g^{-1}[f(x_k)] - x_k\}$. The proper relaxation factor yields a new computed value that is not a full but a partial increment and improves the convergence rate. It should be noted that for the standard GSI, ω is equal to 1, returning the equation to the GSI form noted above. The best practical value of ω moderates the $\{ g^{-1}[f(x_{k+1})] - 1\}$ increment, to increase, or maximize, the convergence rate. This can be realized using some available characteristics of the functions f and g, and either identical or different values of ω may be used in calculating x_{k+1} and y_{k+1}. Shown in figure 2-3, partial relaxation uses the values of x_{k+1} and y_{k+1} moderated by the relaxation factor ω at each iteration step. The successive approximations are much nearer to the solution than the GSI solution of figure 2-2. This is the motivation for exploring relaxation factors as a conditioning method for GSI in the MLS application.

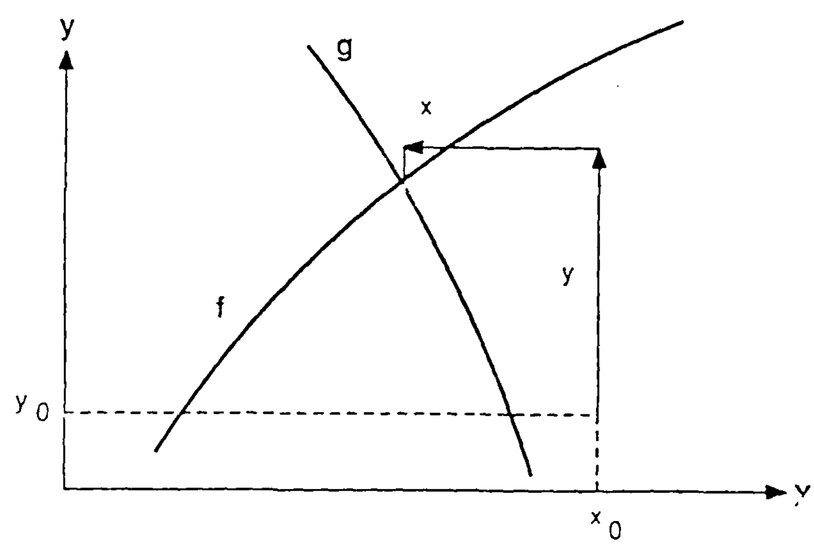

Figure 2-3. The Relaxation Factor Principle

The analogy between the very simple example outlined above and the MLS problem is now presented. The function f is, at least to some extent, similar to the azimuth-angle guidance locus, while the function g is similar to the DME locus. The idea of using relaxation to improve convergence speed and stability is especially attractive since it is a simple improvement to a simple procedure, a quick way without extensive calculation to solve the RNAV problem.

These ideas will now be more formally developed.

SECTION 3

GEOMETRY, NOTATION AND MATHEMATICS

This section presents the geometry, notation and the mathematical foundations for the MLS PRA problem.

The coordinate system for the problem is defined in figure 3-1. The MLS x-axis is selected to be the runway centerline and its extension, with negative values toward the stop-end of the runway. The origin of the coordinate system is the runway threshold, so that a conventional location for the ground equipment has a negative x-value. An azimuth antenna located near the stop-end of a 5000 foot runway has an x-value of approximately $x_A = -5000$, where the subscript A implies azimuth antenna. The positive direction of y lies to the left of an observer who is standing at the origin with the stop-end of the runway behind. The positive direction of z is up. The elevation angle is defined as positive counterclockwise looking along the positive y-axis so that positive angles correspond to positive altitude. Azimuth is defined as positive clockwise from the x-axis, looking down towards the ground. This coordinate system is not right-handed but conforms to that of references 1 and 2. The azimuth antenna boresight is assumed, for simplicity but without loss of generality, to be parallel to the runway centerline; a well-known rotation and de-rotation enable treating other orientations.

Figure 3-1 shows a typical and general geometric situation.

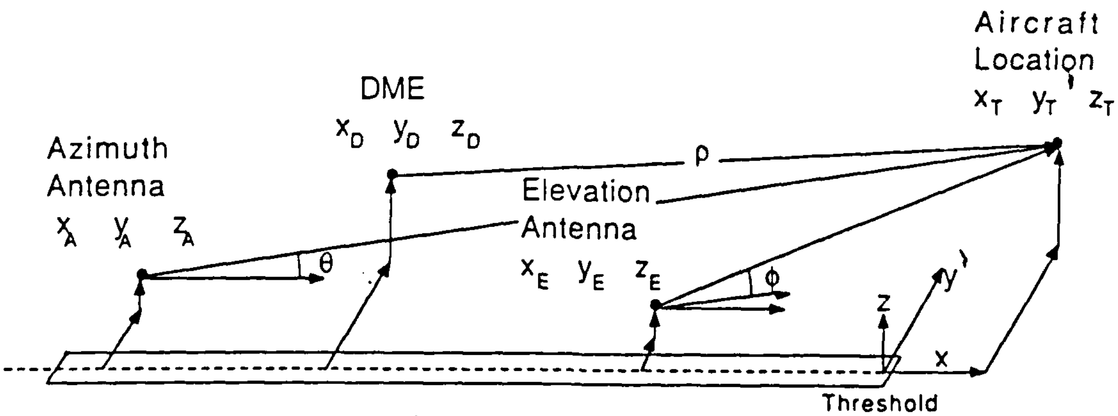

Figure 3-1. Geometry

The ground units' site data, defined below, are transmitted to the aircraft.

x_A, y_A, z_A	Components of position of azimuth antenna
x_D, y_D, z_D	Components of position of DME
x_E, y_E, z_E	Components of position of elevation antenna
x_T, y_T, z_T	Components of true position of the aircraft
x_i, y_i, z_i	Components of the i^{th} estimate, i=0, 1, 2, ..
θ, ϕ	Observed conical azimuth and elevation angles
ρ	Observed slant range from the DME

The observations received in the aircraft, ρ, θ and ϕ, are generated respectively by a DME, a conical-pattern azimuth antenna (AZ), and a conical-pattern elevation antenna (EL).

The observations in the aircraft are now defined.

The observed slant range from the DME to the aircraft is ρ, where:

$$\rho = [(x_T-x_D)^2 + (y_T-y_D)^2 + (z_T-z_D)^2]^{1/2} \tag{3-1}$$

The azimuth antenna forms a conical beam, with the axis of radial symmetry parallel to the y-axis. The observed azimuth angle at the aircraft is θ, measured exterior to the cone, where:

$$\tan\theta = -(y_T-y_A) \;/\; [(x_T-x_A)^2 + (z_T-z_A)^2]^{1/2} \tag{3-2}$$

Similarly, the elevation antenna forms an always-conical beam, with a vertical axis of radial symmetry. The observed elevation angle at the aircraft is ϕ, measured exterior to the cone, where:

$$\tan\phi = (z-z_E) \;/\; [(x-x_E)^2 + (y-y_E)^2]^{1/2} \tag{3-3}$$

The geometrical site data and the observations are combined in the avionics' PRA to determine the aircraft's location.

SECTION 4

A CONVENTIONAL MLS GAUSS-SEIDEL ALGORITHM

A typical GSI MLS PRA is now formed. It will be used as the basis for a successive partially-relaxed algorithm, and will also enable comparisons of performance and complexity. Three distinct topics are addressed in this section:

a. A conventional Gaussian PRA will be defined;

b. It will be shown that this PRA can diverge or converge slowly within the MLS RNAV coverage;

c. The properties of this PRA will be demonstrated analytically.

4.1 DEFINITION

The observations, ρ, θ and ϕ, lead to a GSI PRA which is, in essence, identical to Case 9 of references 2 and 4. Initial conditions for x_0 and y_0 are required, but an initial condition for z_0 is not needed. The conical elevation equation (3-3) solved for altitude, z, is:

$$z_{i+1} = z_E + [(x_i-x_E)^2+(y_i-y_E)^2]^{1/2} \tan\phi \qquad (4\text{-}1)$$

The conical azimuth equation (3-2) solved for lateral position, y is:

$$y_{i+1} = y_A - [(x_i-x_A)^2 + (z_{i+1}-z_A)^2]^{1/2} \tan\theta \qquad (4\text{-}2)$$

The DME equation (3-1) solved for along-runway position, x is:

$$x_{i+1} = x_D + [\rho^2-(z_{i+1}-z_D)^2-(y_{i+1}-y_D)^2]^{1/2} \qquad (4\text{-}3)$$

Iterate to (4-1) until a solution of acceptable accuracy is reached.

4.2 PERFORMANCE

Table 4-1 shows that this PRA can diverge. The four lines at the top of the table show the geometry of the three ground units; the azimuth antenna is in a conventional split-site arrangement, but the DME is collocated with the elevation unit, in accordance with a suggestion in reference 5. Below this area is a line which presents the observations. The four lines below that are organized in columns: the first column shows the three components of the aircraft's true location, the second shows the initial condition assumption, and the other six columns show the estimated position after each of six iterations; the iteration number heads the column. In column 2, the place for z_0 is blank, as that initial condition is not needed. The process is divergent: at each stage the estimates of x and y are further from the true location, although z converges. The initial conditions were selected to be consistent with those used in section 5 and are closer to the solution than the initialization proposed

in reference 2. The configuration in this figure is 3-37 (ground unit arrangement 3, and aircraft location 37), consistent with the database of exercises reported in reference 4; this nomenclature appears in the heading of ground geometry and aircraft position in the tables.

Table 4-1. Divergence with a GSI PRA

GROUND STATION SITE GEOMETRY # 3

AZIMUTH ANTENNA SITE			DME TRANSMITTER SITE			ELEVATION ANTENNA SITE		
X	Y	Z	X	Y	Z	X	Y	Z
-6000.	-1000.	10.	-1000.	500.	5.	-1000.	500.	5.

AIRCRAFT POSITION #37. OBSERVED DATA: RHO=117839.5 THETA=-48.11 PHI= 4.87

	TRUE POS.	INIT. POS.						
ITERATION NUMBER I			1	2	3	4	5	6
X	75000.00	77394.36	71792.68	78990.68	69467.80	81666.87	65333.59	86032.42
Y	90000.00	86410.38	92637.72	86451.89	94417.73	83881.29	97381.91	79313.75
Z	10000.00		9905.31	10000.68	10000.00	10000.00	10000.00	10000.00

Table 4-2, (configuration 2-39), organized in the same format, shows an example of slow convergence with this algorithm. The ground units are arranged in a conventional split-site; the DME is collocated with the azimuth unit. The initialization again uses the procedure consistent with the very accurate initialization used in section 5. After five iterations, the error in lateral position is 118.22′. This error is excessive according to the criterion for slow convergence and excessive error set forth below.

Table 4-2. Slow Convergence with a GSI PRA

GROUND STATION SITE GEOMETRY # 2

AZIMUTH ANTENNA SITE			DME TRANSMITTER SITE			ELEVATION ANTENNA SITE		
X	Y	Z	X	Y	Z	X	Y	Z
-6000.	0.	5.	-6000.	0.	5.	-1000.	500.	5.

AIRCRAFT POSITION #39. OBSERVED DATA: RHO=110841.8 THETA=-42.58 PHI= 5.37

	TRUE POS.	INIT. POS.						
ITERATION NUMBER I			1	2	3	4	5	6
X	75000.00	75256.77	74783.51	75182.17	74846.01	75129.68	74890.45	75092.30
Y	75000.00	74671.41	75233.71	74802.66	75166.07	74859.63	75118.22	74900.13
Z	10000.00		9995.69	10000.89	9999.28	10000.61	9999.48	10000.00

An allowance of 0.017° is given in reference 1 for the "Path Following Error" which may be tolerated in the avionics. There are many sources of error in the avionics; some are governed by physical considerations such as received signal power which impose limits on performance. But, in principle, the algorithm can yield an almost-perfect result. The algorithm, for this discussion, is therefore allowed an error of 0.017°/3, which is negligible when added root sum square (RSS); this allowance is approximately 0.01%, and the value (x_T/10,000) is used hereafter. The algorithm is considered to be slow if the magnitude of the error in any variable exceeds (x_T/10,000) after five iterations.

The examples shown in these tables are not exceptional; they are intended to show that a conventional GSI may have unsatisfactory behavior in geometric situations which might be expected in MLS RNAV.

4.3 ANALYSIS

A rigorous analysis of the stability and speed of convergence of a Gauss-Seidel MLS algorithm can be found in reference 6.

The procedure is outlined:

(a) The method of analysis for GSI algorithms assumes that the functions are linear; therefore (4-1) through (4-3) are linearized about the solution at x_T, y_T, z_T;

(b) Two matrices are formed: A matrix S is the coefficient of the (i+1) values of the variables, while a matrix T is the coefficient of the (i) values of the variables;

(c) The eigenvalues of the matrix-product ($S^{-1}T$) are determined. These are the values of λ for which the determinant $\Delta = |I\lambda - S^{-1}T|$ is zero, where I is the identity matrix in three variables.

As iteration continues, the error at each iteration approaches a change by the factor λ', where λ' is the eigenvalue of greatest magnitude. If $0 > \lambda' > -1$, the sign of the error oscillates , while the magnitude decreases in value at each iteration; if $1 > \lambda' > 0$ the sign of the error remains the same, while the magnitude decreases in value at each iteration; if $|\lambda'| = 1$, the iteration will not converge, while if $|\lambda'| > 1$ the iteration diverges.

Following the procedure outlined above, find the eigenvalues of the $S^{-1}T$ matrix. A simplification is useful in order to enable physical interpretation of this result. Assume a configuration with the three ground units collocated at the origin; then $x_D = y_D = z_D = 0$, $x_A = y_A = z_A = 0$, and $x_E = y_E = z_E = 0$.

The solutions are:

$$\lambda = 0, \ \lambda = -\tan^2\theta, \text{ and } \lambda = -\tan^2\phi. \tag{4-4}$$

The iteration becomes unstable if either angle exceeds 45 degrees. Elevation angles above 30 degrees are out of coverage, but, as noted above, the azimuth angle may be large enough to cause divergence.

If (4-1) is replaced by:

$$z_{i+1} = z_E+\{[(x_i-x_E)^2+(y_i-y_E)^2+(z_i-z_E)^2]^{1/2}\}\sin\phi \tag{4-5}$$

the three eigenvalues of (4-18) become:

$$\lambda = 0, \ \lambda = -\tan^2\theta, \ \lambda = -\sin^2\phi \tag{4-6}$$

This converges more rapidly for large elevation angles. However, it diverges for large azimuth angles.

SECTION 5

A SUCCESSIVE PARTIAL-RELAXATION (SPR) GAUSS-SEIDEL MLS ALGORITHM

The GSI presented in section 4 is now used as the basis for developing the SPR GSI in this section. This section has three parts in which the topics of the definition of the SPR, its performance, and its analysis are presented.

5.1 DEFINITION

The GSI presented above is now used as the basis for development of the SPR; it was defined in section 4 by the relationships:

Conical elevation equation (3-3) solved for altitude, z:

$$z_{i+1} = z_E + [(x_i-x_E)^2+(y_i-y_E)^2]^{1/2} \tan\phi \qquad (5\text{-}1)$$

Conical azimuth equation (3-2) solved for lateral position, y:

$$y_{i+1} = y_A - [(x_i-x_A)^2+(z_{i+1}-z_A)^2]^{1/2} \tan\theta \qquad (5\text{-}2)$$

DME equation (3-1) solved for along-runway position, x:

$$x_{i+1} = x_D + [\rho^2-(z_{i+1}-z_D)^2-(y_{i+1}-y_D)^2]^{1/2} \qquad (5\text{-}3)$$

Iterate to (5-1) until a solution of acceptable accuracy is reached.

The SPR exhibits better convergence for high elevation angles, independent of azimuth, if (5-1) is replaced by:

$$z_{i+1} = z_E + [(x_i-x_E)^2+(y_i-y_E)^2+(z_i-z_E)^2]^{1/2} \sin\phi \qquad (5\text{-}4)$$

which may be derived from (5-1) or the geometry. This formulation is used to generate the numerical results and in the convergence rate analysis.

The eigenvalues of the $S^{-1}T$ matrix are conclusive evidence of the convergence rate for the SPR and therefore the obvious way to find a practical value for the relaxation factor, ω. In order to find a good practical value for ω, we shall examine the eigenvalues and try to reduce them. As an experiment, we consider a separate and different relaxation factor for the x and y equations, (5-2) and (5-3). Insert the relaxation factors, called ω_x and ω_y. Using ω_x and ω_y separately allows the flexibility to develop them differently. The equations, including the relaxation factors, are:

$$x_{i+1} = (1-\omega_x)x_i - \omega_x\{x_D + [\rho^2-(y_{i+1}-y_D)^2-(z_{i+1}-z_D)^2]^{1/2}\} \qquad (5\text{-}5)$$

$$y_{i+1} = (1-\omega_y)y_i - \omega_y\{y_A - [(x_i-x_A)^2+(z_{i+1}-z_A)^2]^{1/2} \tan\theta\} \qquad (5\text{-}6)$$

$$z_{i+1} = z_E + [(x_i-x_E)^2+(y_i-y_E)^2]^{1/2} \tan\phi \qquad (5\text{-}7)$$

Linearize equations (5-5) and (5-6) about the solution, x_T y_T z_T, using a multidimensional Taylor series. Assuming $\phi = z_T = z_A = z_D = z_E = 0$, reduce the equations to two variables and place in matrix form. The resultant simplified matrices are:

$$S= \begin{pmatrix} 1 & \omega_x(y_T-y_D)/R_D \\ 0 & 1 \end{pmatrix} \qquad (5\text{-}8)$$

$$T= \begin{pmatrix} (1-\omega_x) & 0 \\ -\omega_y(\tan\theta(x_T-x_A)/R_A) & (1-\omega_y) \end{pmatrix} \qquad (5\text{-}9)$$

Using equations (5-2) and (5-3) and reducing the factors R_A and R_D results in:

$$R_A = -(y_T-y_A)/\tan\theta$$
$$R_D = x_T-x_D$$

Invert the simplified S matrix and post multiply by the T matrix. The coefficients of the $S^{-1}T$ matrix are:

$$(S^{-1}T)_{11} = (1-\omega_x)-\omega_y\omega_x\tan^2\theta(y_T-y_D)(x_T-x_A)\ /(y_T-y_A)(x_T-x_D) \qquad (5\text{-}10)$$

$$(S^{-1}T)_{12} = -(1-\omega_y)\omega_x(y_T-y_D)/(x_T-x_D) \qquad (5\text{-}11)$$

$$(S^{-1}T)_{21} = \omega_y\tan^2\theta(x_T-x_A)/(y_T-y_A) \qquad (5\text{-}12)$$

$$(S^{-1}T)_{22} = (1-\omega_y) \qquad (5\text{-}13)$$

Again assume collocation at $x = 0$, $y = 0$ and set $\omega_x = 1$; this returns the x equation, (5-5), to the GSI form described above, in (5-3). The resultant matrix is:

$$S^{-1}T = \begin{pmatrix} -\omega_y\tan^2\theta & -(1-\omega_y)(y_T/x_T) \\ \omega_y\tan^2\theta\ (x_T/y_T) & (1-\omega_y) \end{pmatrix} \qquad (5\text{-}14)$$

The eigenvalue equation of the simplified $S^{-1}T$ matrix is:

$$\lambda^2 + \lambda(\omega_y\tan^2\theta-1+\omega_y) = 0 \qquad (5\text{-}15)$$

When the relaxation factor, ω_y, is $\cos^2\theta$, in the collocated two dimensional case, both eigenvalues are zero. This indicates that under the given conditions the error in iteration is zero; in other words, the solution converges exactly on the first iteration. The real situation is not this simple.

Using $\omega_y = \cos^2\theta$ works well in the collocated configuration; however, in the non-collocated configuration this ω_y is not adequate. To generalize the

value of ω_y to non-collocated cases, reexamine equations (5-10) to (5-13). All x dependent variables return to their non-collocated form; $S^{-1}T_{11}$ is multiplied by $(x_T-x_A)/(x_T-x_D)$, x_T-x_D replaces x_T in $S^{-1}T_{12}$ and x_T-x_A replaces x_T in $S^{-1}T_{21}$ of (5-14). More variation is, in general, expected in the x dimension of ground station locations; this direction was therefore selected as the non-collocated dimension. The y dependent variables and ω_x remain as in (5-14). Reexamine the $S^{-1}T$ matrix of (5-14) with the above changes; the eigenvalue equation is:

$$\lambda^2 + \lambda\{\omega_y[(x_T-x_A)/(x_T-x_D)]\tan^2\theta-1+\omega_y\} = 0 \qquad (5\text{-}16)$$

Let:

$$C = (x_T-x_A)/(x_T-x_D)$$

Then (5-16) reduces to:

$$\lambda = -\omega_y(C\ \tan^2\theta+1)+1 \qquad (5\text{-}17)$$

Constrain this to enforce $\lambda = 0$; then :

$$\omega_y = \cos^2\theta/[C+(1-C)\cos^2\theta] \qquad (5\text{-}18)$$

The value C, as well as the new ω_y, are included in the iterative loop and recalculated as x,y,z are being recalculated. The first value of C, and therefore ω_y, is calculated using the x initial condition. Good initial conditions are more important to the SPR than to many other PRAs. The x initial condition of the SPR is used in three equations, which includes a double dependence in the y variable. The initial conditions used are:

$$x = x_D + \rho\cos\theta\ \cos\phi \qquad (5\text{-}19)$$
$$y = y_A - \rho\sin\theta\ \cos\phi \qquad (5\text{-}20)$$
$$z = z_E + \rho\sin\phi \qquad (5\text{-}21)$$

The SPR requires an initial z unlike the GSI, as explained in section 4.1. Through experimentation it was determined that, considering all possible geometries while retaining computational ease, these initial conditions appear to be the best. Other initial conditions may work well in a specific configuration; to generalize the algorithm these were chosen. The improved speed of the SPR algorithm justifies the more complex initial conditions.

5.2 PERFORMANCE

The performance of the SPR is now discussed.

In section 4, tables 4-1 and 4-2 showed that the GSI may diverge or be slow to converge within the coverage of the MLS. Tables 5-1 and 5-2, below, show the behavior of the SPR for the same conditions. A variety of other cases are presented in appendix C, which contains an abbreviated sample of the exercise of this algorithm. All SPR examples and samples were calculated

using the computer code shown in figure A-1.

In tables 5-1 and 5-2, it is evident that there is no sign of either slow convergence or of divergence.

Table 5-1. SPR in Table 4-1 Case

GROUND STATION SITE GEOMETRY # 3

AZIMUTH ANTENNA SITE			DME TRANSMITTER SITE			ELEVATION ANTENNA SITE		
X	Y	Z	X	Y	Z	X	Y	Z
-6000.	-1000.	10.	-1000.	500.	5.	-1000.	500.	5.

AIRCRAFT POSITION #37. OBSERVED DATA: RHO=117839.5 THETA=-48.11 PHI= 4.87

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	75000.00	77394.36	76069.02	75009.83	75000.10	75000.00	75000.00	75000.00
Y	90000.00	86410.38	89091.68	89991.65	89999.91	90000.00	90000.00	90000.00
Z	10000.00	10000.00	9905.99	10000.00	10000.00	10000.00	10000.00	10000.00

Table 5-3 shows the behavior of the SPR PRA in a case involving a large azimuth angle. The GSI PRA of section 4 was divergent for this case, using the same initial conditions.

Table 5-2. SPR in Table 4-2 Case

GROUND STATION SITE GEOMETRY # 2

AZIMUTH ANTENNA SITE			DME TRANSMITTER SITE			ELEVATION ANTENNA SITE		
X	Y	Z	X	Y	Z	X	Y	Z
-6000.	0.	5.	-6000.	0.	5.	-1000.	500.	5.

AIRCRAFT POSITION #39. OBSERVED DATA: RHO=110841.8 THETA=-42.58 PHI= 5.37

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	75000.00	75256.77	75021.91	74999.96	75000.00	75000.00	75000.00	75000.00
Y	75000.00	74671.41	74976.47	75000.05	75000.01	75000.00	75000.00	75000.00
Z	10000.00	10369.21	9999.02	9999.92	10000.00	10000.00	10000.00	10000.00

Table 5-3. Behavior of the SPR PRA with a Large Azimuth Angle

GROUND STATION SITE GEOMETRY # 2

AZIMUTH ANTENNA SITE			DME TRANSMITTER SITE			ELEVATION ANTENNA SITE		
X	Y	Z	X	Y	Z	X	Y	Z
-6000.	0.	5.	-6000.	0.	5.	-1000.	500.	5.

AIRCRAFT POSITION #12. OBSERVED DATA: RHO=25631.8 THETA =-51.29 PHI=2.54

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	10000.00	10015.10	10005.83	10000.00	10000.00	10000.00	10000.00	10000.00
Y	20000.00	19980.28	19995.34	20000.00	20000.00	20000.00	20000.00	20000.00
Z	1000.00	1143.01	999.87	999.95	1000.00	1000.00	1000.00	1000.00

5.3 ANALYSIS

This subsection presents the analysis of the stability characteristics of the SPR. It follows the form of section 4.3, and uses the same mathematical procedures. The analysis of the SPR is identical to the analysis of the GSI with the addition of the relaxation factor, ω_y.

Using (5-7), the tangent form of the altitude equation, shown in (5-1), in order to show the key difference between this SPR PRA and the GSI PRA of section 4, the complete matrices S and T may now be defined:

$$S = \begin{pmatrix} 1 & (y_T-y_D)/R_D & (z_T-z_D)/R_D \\ 0 & 1 & \omega_y(\tan\theta)(z_T-z_A)/R_A \\ 0 & 0 & 1 \end{pmatrix} \quad (5\text{-}22)$$

$$T = \begin{pmatrix} 0 & 0 & 0 \\ -\omega_y(\tan\theta)(x_T-x_A)/R_A & (1-\omega_y) & 0 \\ (\tan\phi)(x_T-x_E)/R_E & (\tan\phi)(y_T-y_E)/R_E & 0 \end{pmatrix} \quad (5\text{-}23)$$

where:

$$\begin{aligned} R_E &= [(x_T-x_E)^2+(y_T-y_E)^2]^{1/2} \\ R_A &= [(x_T-x_A)^2+(z_T-z_A)^2]^{1/2} \\ R_D &= [\rho^2-(y_T-y_D)^2-(z_T-z_D)^2]^{1/2} \end{aligned}$$

The eigenvalue equation for the SPR is:

$$\Delta = \lambda\{\lambda^2 + \lambda[s_{12}t_{21}-s_{12}s_{23}t_{31}+s_{13}t_{31}-t_{22}+t_{32}s_{23}] + s_{13}(t_{21}t_{32}-t_{22}t_{31})\} = 0 \quad (5\text{-}24)$$

One eigenvalue of (5-24) is zero. The others are found from the quadratic term in (5-24), $\{\lambda^2 + B\lambda + C\}$, which is now examined.

Even when several ground units are collocated at the origin, the quadratic term of the determinant is not readily solved. To eliminate the errors inherent in such calculations, computer routines were used to complete the analysis. A simple FORTRAN code was used to enable evaluation of the eigenvalues of (5-24). The eigenvalue behavior was examined in a wide range of situations which are representative of the required coverage. The analysis found one eigenvalue of (5-24) always equal to zero, as expected. The largest magnitude eigenvalue encountered in coverage was approximately 0.2, using the sine-form of the elevation equation, (5-4). The meaning of this result is that the convergence rate, which is mainly determined by the eigenvalue of greatest magnitude, is less than or equal to 1/5, within coverage. The difference between the true location and the estimated location, the error, at each step, is less than or equal to one fifth of the error found in the previous step. By five iterations, the error is reduced by the factor $0.2^5 = 0.00032$.

SECTION 6

COMPARISONS OF PERFORMANCE

Table 6-1 compares various statistical elements of the characteristics and performance of this SPR PRA, the GSI presented in section 4, a rotated GSI (RGSI), and an equivalent Newton-Raphson approach.

Table 6-1. Comparison of Various PRAs

	#1	#2	#3	#4
Property	SPR	GSI*	RGSI	NRA
Lines(less declarations)	25	35	44	48
Mean Iterations	2.37	4.18*	2.72	2.00
Std. Dev., Iterations	1.07	1.48*	0.88	0.56
* or + in Loop	15	9	22	65
* or + outside	6	3	7	3
Transcends. in Loop	3	1	4	7
Transcends. outside	5	2	3	3
No. of * or + to Mean iteration + σ**	134	92*	173	210

* Note that the GSI diverges or is slow in 140 of the 250 cases in the database in reference 4.

** Transcendentals are counted as 5 (* or +), giving total in * or + operations

A Newton-Raphson PRA was presented in reference 4 as an alternate to the algorithm offered in reference 2 as Case 12, because it combines compactness, performance, and a relatively small computational burden. It is used here for comparisons, and, here and in reference 4, is designated as Newton-Raphson Alternate (NRA).

The RGSI was presented in reference 6 as an alternate to the GSI in reference 2. It is used for comparisons. RGSI is basically a GSI in an optimally rotated frame. See reference 6 for a detailed explanation of its principles.

However, these algorithms used different initializations in the various sources referenced; as initialization is an integral and important part of the SPR algorithm, its calculation is included in the multiplication count described below. The initialization used in the SPR is found in section 5. The initial values of x_0 and y_0 are required for the GSI, while the NRA, RGSI and the proposed SPR also require z_0. See

is found in section 5. The initial values of x_0 and y_0 are required for the GSI, while the NRA, RGSI and the proposed SPR also require z_0. See also reference 2. The comparisons are in terms of:

a. Lines of FORTRAN 77 code, as a measure of the required storage;

b. The mean and standard deviation of the number of iterations to converge to the accuracy criterion (x_T/10,000) cited above in section 3, as a measure of speed;

c. The number of products and quotients, and the number of transcendental operations (trigonometric and square-root), in subroutine calculation as measures of the complexity.

d. The number of products, quotients and transcendental operations (as above) outside of subroutine calculation to compensate for different initial conditions.

The SPR (#1) requires less storage than any of the others. It converges slightly faster than the RGSI (#3), and much faster than the GSI (#2), wherever GSI (#2) converges. It is important to recall that a principal motivation for this study was that the GSI (#2) diverges within the MLS coverage, as was demonstrated in table 4-1.

The SPR (#1) has about twice as many products or quotients in the iterative loop as the GSI (#2), but has less than 1/4 of these operations compared to the NRA (#4). The number of transcendental operations in the iterative loops differ for the three PRAs. If it is assumed that a transcendental operation is equivalent to five products, as above, then, within the loop the GSI (#2) has 14 products, the RGSI (#3) has approximately 42 products the NRA (#4) has 75 products and the SPR (#1) has 30 products. The product of the mean number of iterations to convergence plus one σ with the number of products, plus the number of products outside the loop may be taken as a rough order or magnitude measure of the computational burden, as shown in the bottom row of table 6-1. From this viewpoint, the iterative computational burden of the SPR (#1) is about 1.4 times that of the GSI (#2), but only about 0.6 that of the NRA (#4) and 0.77 that of the RGSI (#3). Other Newton-Raphson formulations exist, as in references 2 and 4; the NRA (#4) is the smallest of these in terms of size, and the number of operations, false solutions and singularities.

The data presented in this table show that the SPR (#1) is clearly superior to the equivalent GSI (#2), as the GSI (#2) diverges, or converges slowly, in many areas within the MLS coverage. Further, the SPR (#3) has a smaller computational burden than the NRA (#4) or the RGSI (#3), while its convergence speed is almost equal to that of either. It thus appears to be superior to either.

LIST OF REFERENCES

1. International Civil Aeronautics Association, October 1987, "International Standards, Recommended Practices and Procedures for Air Navigation Services; Aeronautical Communications, Annex 10," Fourth Edition of Volume I, April, 1985, (Correction as of 22 October, 1987), Montreal, Canada.

2. Radio Technical Commission for Aeronautics, 18 March 1988, "Minimum Operational Performance Standards for Airborne MLS Area Navigation Equipment," RTCA/DO-198, Appendix D, Washington, D.C.

3. Strang, G., 1980, Linear Algebra and Its Applications, Second Edition, New York, NY: Academic Press, page 380.

4. Hall, J. W., P. M. Hatzis, and F. D. Powell, June 1990, Examination of the RTCA/DO-198 Position Reconstruction Algorithms for Area Navigation with the Microwave Landing System, ESD-TR-90-308, AD A224 805, Electronic Systems Division, AFSC, Hanscom Air Force Base, MA.

5. United States Department of Transportation, Federal Aviation Agency, Program Engineering and Maintenance Service, October 1987, "Introduction to MLS," Washington, D.C.

6. Hall, J. W., P. M. Hatzis, and F. D. Powell, June 1990, A Gaussian Algorithm Using Coordinate Rotation for Area Navigation Operations with the Microwave Landing System, ESD-TR-90-315, AD A225 642, Electronics Systems Division, AFSC, Hanscom Air Force Base, MA.

APPENDIX A

This section contains the computer code used to generate all example exercises of the SPR contained in this report.

FORTRAN 77 COMPUTER CODE

```
      SUBROUTINE SPR(XA,YA,ZA,XD,YD,ZD,XE,YE,ZE,RHO,THET,PHI
     1  ,ITER,ARAOUT)
      DIMENSION ARAOUT(3,10)
C     ALGORITHM BEGINS HERE.
C     TRIG FUNCTION ABREVIATIONS
      SINP=SIN(PHI)
      COSP=COS(PHI)
      COST=COS(THET)
      TANT=TAN(THET)
C     INITIALIZATION.
      X=XD+RHO*COST*COSP
      Y=YA-RHO*SIN(THET)*COSP
      Z=ZE+RHO*SINP
      WYS=COST**2
C     STORE ESTIMATES, OUTPUT ONLY, NOT NECESSARY TO ALGORITHM
      ARAOUT(1,2)=X
      ARAOUT(2,2)=Y
      ARAOUT(3,2)=Z
C     START OF ITERATION LOOP
      DO 10 I=1,ITER
C     RELAXATION CONSTANT.
      C=ABS(X-XA)/(ABS(X-XD)+0.0001)
      WY=WYS/(C+(1.0-C)*WYS)
C     ITERATION OF X,Y,Z
      Z=ZE+SINP*SQRT((X-XE)**2+(Y-YE)**2+(Z-ZE)**2)
      Y=(1.0-WY)*Y+WY*(YA-TANT*SQRT((X-XA)**2+(Z-ZA)**2))
      R=RHO**2-(Y-YD)**2-(Z-ZD)**2
C     CHECK FOR NEGATIVE RADICAND
      IF(R.LT.0.0)THEN
       X=0.0
       Y=0.0
       Z=0.0
       ELSE
       X=XD+SQRT(R)
```

Figure A-1. Computer Code

```
      ENDIF
C     FOR OUTPUT PURPOSES ONLY
      ARAOUT(1,(2+I))=X
      ARAOUT(2,(2+I))=Y
      ARAOUT(3,(2+I))=Z
C     THIS COMPLETES THE ITERATION.
 10   CONTINUE
      RETURN
      END
```

Figure A-1. Computer Code (concluded)

APPENDIX B

THE SINE FORM OF THE SPR

This form yields a slight, but worthwhile, improvement of the speed of convergence of the SPR. Instead of (5-1), use (5-4), rewritten as:

$$z_{i+1} = z_E + [(x_i - x_E)^2 + (y_i - y_E)^2 + (z_i - z_E)^2]^{1/2} \sin\phi \qquad \text{(C-1)}$$

The matrix S is unchanged, but the third column of matrix T becomes $(\,0 \quad 0 \quad t_{33})^T$ where:

$$t_{33} = [(z_T - z_E)\sin\phi / R_E] \qquad \text{(C-2)}$$

and tan ϕ is replaced by sin ϕ in elements t_{31} and t_{32}. R_E is as defined as:

$$R_E = [(x_T - x_E)^2 + (y_T - y_E)^2 + (z_T - z_E)^2]^{1/2}$$

The characteristic equation, replacing (5-24), is:

$$\Delta = \lambda\{\lambda^2 + \lambda[s_{12}t_{21} - s_{12}s_{23}t_{31} + s_{13}t_{31} - t_{22} + t_{32}s_{23} - t_{33}] +$$

$$s_{13}(t_{21}t_{32} - t_{22}t_{31}) + t_{33}(t_{22} - s_{12}t_{21})\} = 0 \qquad \text{(C-4)}$$

The eigenvalue equation that results using the simplifying assumption that the three ground units are collocated at the origin is still complicated, and was solved, as noted before, by computer. The largest eigenvalue was, as stated before, approximately 0.2 at the extreme elevation of coverage. Eigenvalues for specific examples are included in appendix C.

APPENDIX C

EXERCISE OF THE SPR

The form of this algorithm, defined in appendix B, was tested and exercised in a simulation. Five different arrangements of the ground units were selected. The first assumes that all units are collocated at the usual site for the elevation antenna, the second assumes a conventional split-site with the DME collocated with the azimuth antenna at the usual location of the latter, and the third assumes a split-site but with the DME collocated with the elevation antenna as suggested in reference 5. The fourth and fifth arrangements assume that all three units are separated, in order to test the ability of the algorithm to handle general sites correctly.

The aircraft was assumed to be at 50 different locations which span the possibilities of range, azimuth and elevation conditions within the MLS coverage. The algorithm was tested at each aircraft location for all five of the ground unit arrangements, so that there is a total of 250 tests.

A small sample of these tests is presented in this appendix, consisting of ten aircraft locations at each of the five ground unit arrangements. The structure of the tables is the same as in tables 1 through 4. The heading for each page gives the positions of the ground units, and the five sets below present the components of aircraft location estimate for five different aircraft locations. The first column in each set is the true location of the aircraft, the second column is the initial condition estimate, and the following six columns show the behavior of the algorithm at six successive iterations. The error may be determined in any case by comparing the estimated location after an iteration with the true location in column 1.

Several comments on the characteristics of the algorithm are offered. In the entire database of 250 tests, there is no case in which the negative-radicand condition was encountered, although this was common with the GSI; see the last several lines of code in appendix A. There is no case in which the algorithm diverged, nor any in which it required more than five iterations to meet the accuracy criterion of $|\varepsilon|_{max} < x_T/10{,}000$.

The initial conditions for the database are:

$$x_0 = x_D + \rho\cos\theta\,\cos\phi \tag{C-1}$$
$$y_0 = y_A - \rho\sin\theta\,\cos\phi \tag{C-2}$$
$$z_0 = z_E + \rho\sin\phi \tag{C-3}$$

Table C-1. SPR Algorithm Exercise

GROUND STATION SITE GEOMETRY # 1

AZIMUTH ANTENNA SITE			DME TRANSMITTER SITE			ELEVATION ANTENNA SITE		
X	Y	Z	X	Y	Z	X	Y	Z
-1000.	500.	5.	-1000.	500.	5.	-1000.	500.	5.

AIRCRAFT POSITION #1. OBSERVED DATA: RHO=11628.4 THETA=-54.78 PHI=14.93

EIGENVALUES OF $S^{-1}T$ (-0.0663, 0.0000) (0.0000, 0.0000) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	5000.00	5479.73	5016.08	5000.02	5000.00	5000.00	5000.00	5000.00
Y	10000.00	9679.50	9989.82	9999.99	10000.00	10000.00	10000.00	10000.00
Z	3000.00	3000.00	3000.00	3000.00	3000.00	3000.00	3000.00	3000.00

AIRCRAFT POSITION #6. OBSERVED DATA: RHO=6102.5 THETA= -4.70 PHI=9.38

EIGENVALUES OF $S^{-1}T$ (-0.0266, 0.0000) (0.0000, 0.0000) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	5000.00	5000.55	4000.00	5000.00	5000.00	5000.00	5000.00	5000.00
Y	1000.00	993.31	1000.00	1000.00	1000.00	1000.00	1000.00	1000.00
Z	1000.00	1000.00	1000.00	1000.00	1000.00	1000.00	1000.00	1000.00

AIRCRAFT POSITION #11. OBSERVED DATA: RHO=22742.3 THETA=-59.03 PHI=10.12

EIGENVALUES OF $S^{-1}T$ (-0.0309, 0.0000) (0.0000, 0.0000) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	10000.00	10521.01	10011.12	10000.00	10000.00	10000.00	10000.00	10000.00
Y	20000.00	19696.78	19993.72	20000.00	20000.00	20000.00	20000.00	20000.00
Z	4000.00	4000.00	4000.00	4000.00	4000.00	4000.00	4000.00	4000.00

AIRCRAFT POSITION #16. OBSERVED DATA: RHO=11926.4 THETA=-22.17 PHI=4.79

EIGENVALUES OF $S^{-1}T$ (-0.0696, 0.0000) (0.0000, 0.0000) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	10000.00	10006.40	10000.00	10000.00	10000.00	10000.00	10000.00	10000.00
Y	5000.00	4984.31	5000.00	5000.00	5000.00	5000.00	5000.00	5000.00
Z	1000.00	1000.00	1000.00	1000.00	1000.00	1000.00	1000.00	1000.00

AIRCRAFT POSITION #21. OBSERVED DATA: RHO=47552.1 THETA=-56.17 PHI=6.03

EIGENVALUES OF $S^{-1}T$ (-0.0110, 0.0000) (0.0000, 0.0000) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	25000.00	25328.99	25002.02	25000.00	25000.00	25000.00	25000.00	25000.00
Y	40000.00	39781.48	39998.67	40000.00	40000.00	40000.00	40000.00	40000.00
Z	5000.00	5000.00	5000.00	5000.00	5000.00	5000.00	5000.00	5000.00

Table C-1. SPR Algorithm Exercise (continued)

GROUND STATION SITE GEOMETRY # 1

AZIMUTH ANTENNA SITE			DME TRANSMITTER SITE			ELEVATION ANTENNA SITE		
X	Y	Z	X	Y	Z	X	Y	Z
-1000.	500.	5.	-1000.	500.	5.	-1000.	500.	5.

AIRCRAFT POSITION #26. OBSERVED DATA: RHO=27793.4 THETA=-19.99 PHI=5.15

EIGENVALUES OF $S^{-1}T$ (-0.0081, 0.0000) (0.0000, 0.0000) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	25000.00	25013.98	25000.01	25000.00	25000.00	25000.00	25000.00	25000.00
Y	10000.00	9961.64	9999.99	10000.00	10000.00	10000.00	10000.00	10000.00
Z	2500.00	2500.00	2500.00	2500.00	2500.00	2500.00	2500.00	2500.00

AIRCRAFT POSITION #31. OBSERVED DATA: RHO=90835.8 THETA=-55.10 PHI=6.32

EIGENVALUES OF $S^{-1}T$ (-0.0121, 0.0000) (0.0000, 0.0000) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	50000.00	50654.61	50004.08	50000.00	50000.00	50000.00	50000.00	50000.00
Y	75000.00	74547.63	74997.21	75000.00	75000.00	75000.00	75000.00	75000.00
Z	10000.00	10000.00	10000.00	10000.00	10000.00	10000.00	10000.00	10000.00

AIRCRAFT POSITION #37. OBSERVED DATA: RHO=117839.5 THETA=-49.42 PHI=4.87

EIGENVALUES OF $S^{-1}T$ (-0.0072, 0.0000) (0.0000, 0.0000) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	75000.00	75378.19	75000.93	50000.00	50000.00	50000.00	50000.00	50000.00
Y	90000.00	89677.48	89999.21	75000.00	75000.00	75000.00	75000.00	75000.00
Z	10000.00	10000.00	10000.00	10000.00	10000.00	10000.00	10000.00	10000.00

AIRCRAFT POSITION #41. OBSERVED DATA: RHO=129193.8 THETA=-43.85 PHI=8.90

EIGENVALUES OF $S^{-1}T$ (-0.0239, 0.0000) (0.0000, 0.0000) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	90000.00	91048.19	90005.83	90000.00	90000.00	90000.00	90000.00	90000.00
Y	90000.00	88921.61	89994.07	90000.00	90000.00	90000.00	90000.00	90000.00
Z	20000.00	20000.00	20000.00	20000.00	20000.00	20000.00	20000.00	20000.00

AIRCRAFT POSITION #46. OBSERVED DATA: RHO=103611.0 THETA=-28.54 PHI=1.10

EIGENVALUES OF $S^{-1}T$ (-0.0004, 0.0000) (0.0000, 0.0000) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	90000.00	90004.98	90000.00	90000.00	90000.00	90000.00	90000.00	90000.00
Y	50000.00	49990.82	50000.00	50000.00	50000.00	50000.00	50000.00	50000.00
Z	2000.00	2000.00	2000.00	2000.00	2000.00	2000.00	2000.00	2000.00

Table C-1. SPR Algorithm Exercise (continued)

GROUND STATION SITE GEOMETRY # 2

AZIMUTH ANTENNA SITE			DME TRANSMITTER SITE			ELEVATION ANTENNA SITE		
X	Y	Z	X	Y	Z	X	Y	Z
-6000.	0.	5.	-6000.	0.	5.	-1000.	500.	5.

AIRCRAFT POSITION #1. OBSERVED DATA: RHO=15164.8 THETA=-41.26 PHI=14.93

EIGENVALUES OF $S^{-1}T$ (-0.0123,-0.1073) (-0.0123, 0.1073) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	5000.00	5015.82	5124.14	5005.65	4998.79	4999.88	5000.01	5000.00
Y	10000.00	9662.63	9861.12	9997.44	10001.51	10000.09	9999.99	10000.00
Z	3000.00	3910.82	3001.82	2987.77	3000.12	3000.12	3000.01	3000.00

AIRCRAFT POSITION #6. OBSERVED DATA: RHO=11090.1 THETA=-5.17 PHI=9.38

EIGENVALUES OF $S^{-1}T$ (-0.0152, 0.0000) (0.0000, 0.0000) (0.0006, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	5000.00	4897.23	4999.57	4999.99	5000.00	5000.00	5000.00	5000.00
Y	1000.00	986.62	990.80	999.89	1000.00	1000.00	1000.00	1000.00
Z	1000.00	1813.23	1013.83	1000.18	1000.00	1000.00	1000.00	1000.00

AIRCRAFT POSITION #11. OBSERVED DATA: RHO=25922.2 THETA=-50.49 PHI=10.12

EIGENVALUES OF $S^{-1}T$ (-0.0080,-0.0722) (-0.0080, 0.0722) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	10000.00	10234.77	10094.26	10000.42	9999.51	9999.99	10000.00	10000.00
Y	20000.00	19689.00	19925.80	20000.33	20000.40	20000.00	20000.00	20000.00
Z	4000.00	4558.60	3992.13	3996.64	3999.98	4000.02	4000.00	4000.00

AIRCRAFT POSITION #16. OBSERVED DATA: RHO=16792.6 THETA=-17.32 PHI=4.79

EIGENVALUES OF $S^{-1}T$ (-0.0023,-0.0112) (-0.0023, 0.0112) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	10000.00	9975.02	10002.63	10000.01	10000.00	10000.00	10000.00	10000.00
Y	5000.00	4982.57	4991.38	4999.98	5000.00	5000.00	5000.00	5000.00
Z	1000.00	1405.97	1000.93	999.94	1000.00	1000.00	1000.00	1000.00

AIRCRAFT POSITION #21. OBSERVED DATA: RHO=50852.2 THETA=-51.87 PHI= 6.03

EIGENVALUES OF $S^{-1}T$ (-0.0041,-0.0316) (-0.0041, 0.0316) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	25000.00	25226.13	25037.29	24999.90	24999.96	25000.00	25000.00	25000.00
Y	40000.00	39778.71	39971.35	40000.13	40000.03	40000.00	40000.00	40000.00
Z	5000.00	5346.66	4997.75	4999.62	5000.00	5000.00	5000.00	5000.00

Table C-1. SPR Algorithm Exercise (continued)

GROUND STATION SITE GEOMETRY # 2

AZIMUTH ANTENNA SITE			DME TRANSMITTER SITE			ELEVATION ANTENNA SITE		
X	Y	Z	X	Y	Z	X	Y	Z
-6000.	0.	5.	-6000.	0.	5.	-1000.	500.	5.

AIRCRAFT POSITION #26. OBSERVED DATA: RHO=32668.4 THETA=-17.82 PHI=5.15
EIGENVALUES OF $S^{-1}T$ (-0.0033,-0.0085) (- 0.0033, 0.0085) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	25000.00	24974.68	25003.55	25000.01	25000.00	25000.00	25000.00	25000.00
Y	10000.00	9959.63	9988.87	9999.99	10000.00	10000.00	10000.00	10000.00
Z	2500.00	2937.62	2500.47	2499.96	2500.00	2500.00	2500.00	2500.00

AIRCRAFT POSITION #31. OBSERVED DATA: RHO=94132.4 THETA=-52.82 PHI=6.32
EIGENVALUES OF $S^{-1}T$ (-0.0052,-0.0247) (- 0.0052, 0.0247) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	50000.00	50539.55	50045.46	49999.75	49999.96	50000.00	50000.00	50000.00
Y	75000.00	74544.59	74966.43	75000.22	75000.02	75000.00	75000.00	75000.00
Z	10000.00	10362.73	9997.01	9999.75	10000.00	10000.00	10000.00	10000.00

AIRCRAFT POSITION #37. OBSERVED DATA: RHO=121494.4 THETA=-47.80 PHI=4.87
EIGENVALUES OF $S^{-1}T$ (-0.0033,-0.0145) (-0.0033, 0.0145) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	75000.00	75320.23	75022.21	74999.94	74999.99	75000.00	75000.00	75000.00
Y	90000.00	89675.67	89980.13	90000.06	90000.01	90000.00	90000.00	90000.00
Z	10000.00	10310.01	9998.96	9999.93	10000.00	10000.00	10000.00	10000.00

AIRCRAFT POSITION #41. OBSERVED DATA: RHO=133100.7 THETA=-42.55 PHI=8.90
EIGENVALUES OF $S^{-1}T$ (-0.0111,-0.0197) (-0.0111, 0.0197) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	90000.00	90878.65	90064.38	89999.69	89999.96	90000.00	90000.00	90000.00
Y	90000.00	88915.59	89932.32	90000.42	90000.42	90000.00	90000.00	90000.00
Z	20000.00	20604.66	19995.39	19999.66	20000.00	20000.00	20000.00	20000.00

AIRCRAFT POSITION #46. OBSERVED DATA: RHO=108258.9 THETA=-27.51 PHI=1.10
EIGENVALUES OF $S^{-1}T$ (-0.0002,-0.0018) (-0.0002, 0.0018) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	90000.00	90002.92	90000.41	90000.00	90000.00	90000.00	90000.00	90000.00
Y	50000.00	49990.73	49999.22	50000.00	50000.00	50000.00	50000.00	50000.00
Z	2000.00	2089.49	2000.00	2000.00	2000.00	2000.00	2000.00	2000.00

Table C-1. SPR Algorithm Exercise (continued)

GROUND STATION SITE GEOMETRY # 3

AZIMUTH ANTENNA SITE			DME TRANSMITTER SITE			ELEVATION ANTENNA SITE		
X	Y	Z	X	Y	Z	X	Y	Z
-6000.	-1000.	10.	-1000.	500.	5.	-1000.	500.	5.

AIRCRAFT POSITION # 1. OBSERVED DATA: RHO=11628.4 THETA=-43.98 PHI=14.93

EIGENVALUES OF $S^{-1}T$ (-0.0663, 0.0000) (0.0000, 0.0000) (0.0860, 0.0000)

	TRUE POS	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	5000.00	7085.41	6625.98	5166.71	5015.13	5001.31	5000.11	5000.01
Y	10000.00	6802.31	8836.54	9892.64	9990.43	9999.17	9999.93	9999.99
Z	3000.00	3000.00	2755.74	3000.00	3000.00	3000.00	3000.00	3000.00

AIRCRAFT POSITION # 6. OBSERVED DATA: RHO= 6102.5 THETA=-10.26 PHI=9.38

EIGENVALUES OF $S^{-1}T$ (-0.0266, 0.0000) (0.0000, 0.0000) (0.0425, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	5000.00	4924.44	5007.27	5000.21	5000.01	5000.00	5000.00	5000.00
Y	1000.00	72.84	934.08	997.50	999.89	1000.00	1000.00	1000.00
Z	1000.00	1000.00	986.98	1000.00	1000.00	1000.00	1000.00	1000.00

AIRCRAFT POSITION # 11. OBSERVED DATA: RHO=22742.3 THETA=-51.86 PHI=10.12

EIGENVALUES OF $S^{-1}T$ (-0.0309, 0.0000) (0.0000, 0.0000) (0.0502, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	10000.00	12826.98	12038.18	10158.57	10008.51	10000.43	10000.02	10000.00
Y	20000.00	16608.65	18743.26	19909.70	19995.20	19999.76	19999.99	20000.00
Z	4000.00	4000.00	3799.65	4000.00	4000.00	4000.00	4000.00	4000.00

AIRCRAFT POSITION # 16. OBSERVED DATA: RHO=11926.4 THETA=-20.52 PHI=4.79

EIGENVALUES OF $S^{-1}T$ (-0.0070, 0.0000) (0.0000, 0.0000) (0.0423, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	10000.00	10130.76	10109.67	10004.68	10000.20	10000.01	10000.00	10000.00
Y	5000.00	3166.07	4730.43	4988.[illegible]4	4999.51	4999.98	5000.00	5000.00
Z	1000.00	1000.00	963.49	1000.00	1000.00	1000.00	1000.00	1000.00

AIRCRAFT POSITION # 21. OBSERVED DATA: RHO=47552.1 THETA=-52.55 PHI=6.03

EIGENVALUES OF $S^{-1}T$ (-0.0110, 0.0000) (0.0000, 0.0000) (0.0245, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	25000.00	27752.35	26661.03	25065.51	25001.67	25000.04	25000.00	25000.00
Y	40000.00	36544.01	38871.02	39956.80	39998.90	39999.97	40000.00	40000.00
Z	5000.00	5000.00	4876.56	5000.00	5000.00	5000.00	5000.00	5000.00

Table C-1. SPR Algorithm Exercise (continued)

GROUND STATION SITE GEOMETRY # 3

AZIMUTH ANTENNA SITE			DME TRANSMITTER SITE			ELEVATION ANTENNA SITE		
X	Y	Z	X	Y	Z	X	Y	Z
-6000.	-1000.	10.	-1000.	500.	5.	-1000.	500.	5.

AIRCRAFT POSITION # 26. OBSERVED DATA: RHO=27793.4 THETA=-19.48 PHI=5.15

EIGENVALUES OF $S^{-1}T$ (-0.0806, 0.0000) (0.0000, 0.0000) (0.0177, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	25000.00	25096.91	25076.44	25000.95	25000.02	25000.00	25000.00	25000.00
Y	10000.00	8230.46	9799.16	9997.39	9999.95	10000.00	10000.00	10000.00
Z	2500.00	2500.00	2458.57	2500.00	2500.00	2500.00	2500.00	2500.00

AIRCRAFT POSITION # 31. OBSERVED DATA: RHO=90835.8 THETA=-53.19 PHI=6.32

EIGENVALUES OF $S^{-1}T$ (-0.0121, 0.0000) (0.0000, 0.0000) (0.0131, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	50000.00	53059.99	51575.04	50024.26	50000.32	50000.00	50000.00	50000.00
Y	75000.00	71280.30	73914.67	74983.39	74999.78	75000.00	75000.00	75000.00
Z	10000.00	10000.00	9869.17	10000.00	10000.00	10000.00	10000.00	10000.00

AIRCRAFT POSITION # 37. OBSERVED DATA: RHO=117839.5 THETA=-48.11 PHI=4.87

EIGENVALUES OF $S^{-1}T$ (-0.0072, 0.0000) (0.0000, 0.0000) (0.0094, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	75000.00	77394.36	76069.02	75009.83	75000.10	75000.00	75000.00	75000.00
Y	90000.00	86410.38	89091.69	89991.65	89999.91	90000.00	90000.00	90000.00
Z	10000.00	10000.00	9905.99	10000.00	10000.00	10000.00	10000.00	10000.00

AIRCRAFT POSITION # 41. OBSERVED DATA: RHO=129193.8 THETA=-42.86 PHI=8.90

EIGENVALUES OF $S^{-1}T$ (-0.0240, 0.0000) (0.0000, 0.0000) (0.0078, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	90000.00	92557.83	90797.13	89993.11	89999.95	90000.00	90000.00	90000.00
Y	90000.00	85822.71	89217.09	90007.00	90000.05	89999.99	89999.99	89999.99
Z	20000.00	20000.00	19844.73	20000.00	20000.00	20000.00	20000.00	20000.00

AIRCRAFT POSITION # 46. OBSERVED DATA: RHO=103611.0 THETA=-27.97 PHI=1.10

EIGENVALUES OF $S^{-1}T$ (-0.0004, 0.0000) (0.0000, 0.0000) (0.0067, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	90000.00	90487.83	90191.16	90001.42	90000.01	90000.00	90000.00	90000.00
Y	50000.00	47592.47	49647.48	49997.39	49999.98	50000.00	50000.00	50000.00
Z	2000.00	2000.00	1986.62	2000.00	2000.00	2000.00	2000.00	2000.00

Table C-1. SPR Algorithm Exercise (continued)

GROUND STATION SITE GEOMETRY # 4

AZIMUTH ANTENNA SITE			DME TRANSMITTER SITE			ELEVATION ANTENNA SITE		
X	Y	Z	X	Y	Z	X	Y	Z
-1000.	-500.	5.	-5000.	-1000.	25.	-1000.	500.	5.

AIRCRAFT POSITION # 1. OBSERVED DATA: RHO=15160.8 THETA=-57.44 PHI=14.93

EIGENVALUES OF $S^{-1}T$ (-0.0248,-0.1065) (-0.0248, 0.1065) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	5000.00	2885.07	5089.40	5046.99	5000.05	4999.53	4999.99	5000.01
Y	10000.00	11846.20	9845.80	9957.82	10000.70	10000.44	10000.00	10000.00
Z	3000.00	3909.80	3253.48	2997.31	2997.23	2999.97	3000.03	3000.00

AIRCRAFT POSITION # 6. OBSERVED DATA: RHO=10244.5 THETA=-13.85 PHI=9.38

EIGENVALUES OF $S^{-1}T$ (-0.0351, 0.0000) (0.0000, 0.0000) (0.0070, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	5000.00	4813.39	5000.43	5000.01	5000.00	5000.00	5000.00	5000.00
Y	1000.00	1920.29	988.93	999.73	999.99	1000.00	1000.00	1000.00
Z	1000.00	1675.36	1018.04	1000.41	1000.01	1000.00	1000.00	1000.00

AIRCRAFT POSITION # 11. OBSERVED DATA: RHO=26111.3 THETA=-60.28 PHI=10.12

EIGENVALUES OF $S^{-1}T$ (-0.0168,-0.0660) (-0.0168, 0.0660) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	10000.00	7744.12	9837.92	10026.50	10000.63	9999.89	10000.00	10000.00
Y	20000.00	21823.73	20089.37	19980.29	19999.66	20000.08	20000.00	20000.00
Z	4000.00	4591.82	4132.92	4003.99	3999.41	3999.98	4000.00	4000.00

AIRCRAFT POSITION # 16. OBSERVED DATA: RHO=16184.9 THETA=-26.47 PHI=4.79

EIGENVALUES OF $S^{-1}T$ (-0.0104, 0.0000) (-0.0086, 0.0000) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	10000.00	9437.46	9989.93	10000.38	10000.00	10000.00	10000.00	10000.00
Y	5000.00	6689.38	5021.25	4999.04	5000.01	5000.00	5000.00	5000.00
Z	1000.00	1355.27	1023.62	1000.06	1000.00	1000.00	1000.00	1000.00

AIRCRAFT POSITION # 21. OBSERVED DATA: RHO=51046.6 THETA=-56.83 PHI=6.03

EIGENVALUES OF $S^{-1}T$ (-0.0085,-0.0270) (-0.0085, 0.0270) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	25000.00	22776.75	24681.78	25010.67	25000.16	24999.99	25000.00	25000.00
Y	40000.00	41990.60	40223.72	39991.94	39999.89	40000.01	40000.00	40000.00
Z	5000.00	5367.07	5059.68	5002.07	4999.93	5000.00	5000.00	5000.00

Table C-1. SPR Algorithm Exercise (continued)

GROUND STATION SITE GEOMETRY # 4

AZIMUTH ANTENNA SITE			DME TRANSMITTER SITE			ELEVATION ANTENNA SITE		
X	Y	Z	X	Y	Z	X	Y	Z
-1000.	-500.	5.	-5000.	-1000.	25.	-1000.	500.	5.

AIRCRAFT POSITION # 26. OBSERVED DATA: RHO=32048.8 THETA=-21.90 PHI=5.15

EIGENVALUES OF $S^{-1}T$ (-0.0082, 0.0000) (-0.0045, 0.0000) (0.0000, 0.0000)

ITERATION NUMBER I	TRUE POS.	INIT. EST.	1	2	3	4	5	6
X	25000.00	24615.96	24984.53	25000.27	25000.00	25000.00	25000.00	25000.00
Y	10000.00	11405.60	10038.16	9999.26	10000.00	10000.00	10000.00	10000.00
Z	2500.00	2882.00	2517.56	2500.02	2500.00	2500.00	2500.00	2500.00

AIRCRAFT POSITION # 31. OBSERVED DATA: RHO=94342.5 THETA=-55.46 PHI= 6.32

EIGENVALUES OF $S^{-1}T$ (-0.0076,-0.0205) (-0.0076, 0.0205) (0.0000, 0.0000)

ITERATION NUMBER I	TRUE POS.	INIT. EST.	1	2	3	4	5	6
X	50000.00	48167.58	49614.10	50008.99	50000.14	50000.00	50000.00	50000.00
Y	75000.00	76739.53	75270.92	74993.30	74999.90	75000.00	75000.00	75000.00
Z	10000.00	10385.85	10052.27	10001.38	9999.97	10000.00	10000.00	10000.00

AIRCRAFT POSITION # 37. OBSERVED DATA: RHO=121575.1 THETA=-49.74 PHI=4.87

EIGENVALUES OF $S^{-1}T$ (-0.0049,-0.0116) (-0.0049, 0.0116) (0.0000, 0.0000)

ITERATION NUMBER I	TRUE POS.	INIT. EST.	1	2	3	4	5	6
X	75000.00	73293.61	74716.30	75003.88	75000.03	75000.01	75000.01	75000.01
Y	90000.00	91935.27	90244.70	89996.54	89999.98	90000.00	90000.00	90000.00
Z	10000.00	10316.85	10035.97	10000.55	9999.99	10000.00	10000.00	10000.00

AIRCRAFT POSITION # 41. OBSERVED DATA: RHO=133060.1 THETA=-44.17 PHI=8.90

EIGENVALUES OF $S^{-1}T$ (-0.0126,-0.0139) (-0.0126, 0.0139) (0.0000, 0.0000)

ITERATION NUMBER I	TRUE POS.	INIT. EST.	1	2	3	4	5	6
X	90000.00	89295.77	89821.35	90006.04	90000.05	90000.00	90000.00	90000.00
Y	90000.00	91092.70	90173.88	89993.58	89999.96	90000.00	90000.00	90000.00
Z	20000.00	20598.38	20055.87	20000.55	19999.98	20000.00	20000.00	20000.00

AIRCRAFT POSITION # 46. OBSERVED DATA: RHO=107842.0 THETA=-29.02 PHI=1.10

EIGENVALUES OF $S^{-1}T$ (-0.0130,-0.0004) (-0.0130, 0.0004) (0.0000, 0.0000)

ITERATION NUMBER I	TRUE POS.	INIT. EST.	1	2	3	4	5	6
X	90000.00	89283.16	89943.52	90000.17	90000.00	90000.00	90000.00	90000.00
Y	50000.00	51809.41	50104.87	49999.68	50000.00	50000.00	50000.00	50000.00
Z	2000.00	2081.47	2004.90	2000.01	2000.00	2000.00	2000.00	2000.00

Table C-1. SPR Algorithm Exercise (continued)

GROUND STATION SITE GEOMETRY # 5

AZIMUTH ANTENNA SITE			DME TRANSMITTER SITE			ELEVATION ANTENNA SITE		
X	Y	Z	X	Y	Z	X	Y	Z
-12000.	1000.	0.	-5000.	-1500.	5.	-1000.	-1000.	10.

AIRCRAFT POSITION # 1. OBSERVED DATA: RHO=15531.3 THETA=-27.54 PHI=13.42

EIGENVALUES OF $S^{-1}T$ (-0.0578,-0.0853) (-0.0578, 0.0853) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	5000.00	8395.82	4146.92	5074.91	5000.52	4999.53	5000.03	5000.00
Y	10000.00	7983.99	10653.92	9919.87	10000.77	10000.42	9999.97	10000.00
Z	3000.00	3614.97	3141.26	3055.07	2995.31	2999.96	3000.03	3000.00

AIRCRAFT POSITION # 6. OBSERVED DATA: RHO=10355.7 THETA= .00 PHI=8.90

EIGENVALUES OF $S^{-1}T$ (-0.0144, 0.0000) (0.0000, 0.0000) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	5000.00	5231.09	4994.70	4999.95	5000.00	5000.00	5000.00	5000.00
Y	1000.00	1000.00	1000.00	1000.00	1000.00	1000.00	1000.00	1000.00
Z	1000.00	1611.50	1051.92	1000.51	1000.00	1000.00	1000.00	1000.00

AIRCRAFT POSITION # 11. OBSERVED DATA: RHO=26518.1 THETA=-40.35 PHI=9.55

EIGENVALUES OF $S^{-1}T$ (-0.0426,-0.0588) (-0.0426, 0.0588) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	10000.00	14927.84	8259.49	10158.55	9997.18	9999.62	10000.03	10000.00
Y	20000.00	17932.80	21082.24	19880.23	20002.70	20000.25	19999.97	20000.00
Z	4000.0	4411.31	4180.92	4044.08	3996.03	4000.07	4000.01	4000.00

AIRCRAFT POSITION # 16. OBSERVED DATA: RHO=16378.0 THETA=-10.29 PHI=4.52

EIGENVALUES OF $S^{-1}T$ (-0.0243, 0.0000) (-0.0091, 0.0000) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	10000.00	11064.32	9937.64	10001.88	9999.95	10000.00	10000.00	10000.00
Y	5000.00	3917.77	5135.76	4995.49	5000.12	5000.00	5000.00	5000.00
Z	1000.00	1300.02	1041.18	1001.14	999.97	1000.00	1000.00	1000.00

AIRCRAFT POSITION # 21. OBSERVED DATA: RHO=51450.9 THETA=-46.25 PHI=5.87

EIGENVALUES OF $S^{-1}T$ (-0.0226,-0.0244) (-0.0226, 0.0244) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	25000.00	30393.50	23114.34	25115.73	24997.49	25000.00	25000.00	25000.00
Y	40000.00	37970.62	41281.59	39914.35	40001.92	40000.00	40000.00	40000.00
Z	5000.00	5270.56	5154.76	5014.44	4999.12	5000.02	5000.00	5000.00

Table C-1. SPR Algorithm Exercise (concluded)

GROUND STATION SITE GEOMETRY # 5

AZIMUTH ANTENNA SITE			DME TRANSMITTER SITE			ELEVATION ANTENNA SITE		
X	Y	Z	X	Y	Z	X	Y	Z
-12000.	1000.	0.	-5000.	-1500.	5.	-1000.	-1000.	10.

AIRCRAFT POSITION # 26. OBSERVED DATA: RHO=32225.4 THETA=-13.64 PHI=5.04

EIGENVALUES OF $S^{-1}T$ (-0.0127,-0.0049) (-0.0127, 0.0049) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	25000.00	26195.23	24927.81	25001.65	24999.97	25000.00	25000.00	25000.00
Y	10000.00	8570.77	10174.65	9995.55	10000.07	10000.00	10000.00	10000.00
Z	2500.00	2841.30	2555.18	2500.62	2499.99	2500.00	2500.00	2500.00

AIRCRAFT POSITION # 31. OBSERVED DATA: RHO=94747.8 THETA=-49.68 PHI=6.23

EIGENVALUES OF $S^{-1}T$ (-0.0156,-0.0218) (-0.0156, 0.0218) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	50000.00	55945.34	47960.60	50088.32	49999.31	49999.97	50000.00	50000.00
Y	75000.00	72813.12	76401.57	74935.27	75000.54	75000.02	75000.00	75000.00
Z	10000.00	10290.61	10186.84	10008.78	9999.62	10000.00	10000.00	10000.00

AIRCRAFT POSITION # 37. OBSERVED DATA: RHO=121951.4 THETA=-45.46 PHI=4.82

EIGENVALUES OF $S^{-1}T$ (-0.0107,-0.0125) (-0.0107, 0.0125) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	75000.00	80230.88	73534.17	75041.38	74999.66	75000.00	75000.00	75000.00
Y	90000.00	87619.88	91245.95	89963.40	90000.30	90000.00	90000.00	90000.00
Z	10000.00	10249.29	10140.14	10003.70	9999.90	10000.00	10000.00	10000.00

AIRCRAFT POSITION # 41. OBSERVED DATA: RHO=133405.6 THETA=-40.57 PHI=8.83

EIGENVALUES OF $S^{-1}T$ (-0.0173,-0.0193) (-0.0173, 0.0193) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	90000.00	95133.26	88970.66	90037.52	89999.98	89999.98	90000.00	90000.00
Y	90000.00	86738.53	91006.21	89960.09	90000.05	90000.02	89999.99	89999.99
Z	20000.00	20486.39	20232.78	20004.26	19999.85	20000.00	20000.00	20000.00

AIRCRAFT POSITION # 46. OBSERVED DATA: RHO=108079.7 THETA=-25.65 PHI=1.09

EIGENVALUES OF $S^{-1}T$ (-0.0099, 0.0004) (-0.0006, 0.0000) (0.0000, 0.0000)

	TRUE POS.	INIT. EST.						
ITERATION NUMBER I			1	2	3	4	5	6
X	90000.00	92407.26	89733.52	90003.09	89999.98	90000.00	90000.00	90000.00
Y	50000.00	47784.69	50487.79	49994.29	50000.05	50000.00	50000.00	50000.00
Z	2000.00	2071.41	2020.29	2000.15	2000.00	2000.00	2000.00	2000.00